Online Appendix to "Supply and Demand in a Two-Sector Matching Model"

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The first part of the Online Appendix, provides additional results for the case of abundant jobs. I look at what happens to wage levels and range if we isolate the increases in spread from the increases in levels; and vice versa. The second part provides a more formal discussion of the cases in which firms have no market power. Finally, the last part features the omitted figures for the examples from the introduction.

Isolated Increases in Surplus Levels and Spread

Isolated Spread

Definition OA1. A (strict) spread of sector one surplus is *isolated* if

$$\pi^1(u^c(\rho_1), P^1(\rho_1), \rho_1) = \pi^1(u^c(\rho_1), P^1(\rho_1), \rho_2).$$

For simplicity, I assume that the surplus function in sector two is strictly supermodular and that jobs are abundant in each sector ($R^1, R^2 \geq 1$).

Proposition OA1. If jobs are abundant in each sector and the surplus function in sector two is strictly supermodular, then an isolated, strict spread of surplus implies that (i) in sector two, wages strictly increase for all agents and the higher the talent the greater the increase; (ii) in sector one, wages strictly increase for a positive mass of the most talented agents and, if the critical ability remains constant, strictly decrease for a positive mass of least talented agents; and (iii) in both sectors wage range increases strictly.

Proof. Abundance of jobs in each sector implies that the least talented, matched agents earn the entire surplus (by Lemma 1); and as their wages have to be equal across sectors (by Lemma 2) we have that:

$$\pi^1(u^c(\rho_k), P^1(\rho_k)) = \pi^2(v^c(\rho_k), P^2(\rho_k)) \qquad \text{for any } k = 1, 2, \tag{OA1}$$

where $P^i(\rho_k) = \frac{R^i - M^i(\rho_k)}{R^i}$. Given that sector one employment expands and sector two employment shrinks, this can hold only if $v^c(\rho_2) \leq v^c(\rho_1)$ and $u^c(\rho_2) \geq u^c(\rho_1)$, as with scarce jobs. Moreover, note that if the change in either $u^c(\rho)$ or $v^c(\rho)$ is strict, then this equation can hold only if the change in masses is strict as well. If both $u^c(\rho)$ and $v^c(\rho)$ remain unchanged, then the strict fall in the quality in $u^c(\rho_1)$'s match – by Theorem 2 (statement (iv)) in Appendix B) – also implies that the change in mass is strict. Thus, as expected, but contrary to the scarce jobs case, employment increases strictly in sector one and falls strictly in sector two.

The analysis for sector two is exactly the same as in the proof of Proposition 7. The only difference is that the improvement in wages for $v > v^c(\rho_1)$ is strict, as the surplus function is supermodular and isolated, strict spread implies that $P^2(\rho_2) > P^2(\rho_1)$. This implies that the increase in top sector one wages is strict as well (see Equation (36)). The direction of change is still ambiguous for bottom wages in general, but not if $u^c(\rho_1) = u^c(\rho_2)$, in which case the wage of the least talented worker depends only on the surplus she produces: $\pi^1(u^c, P^1)$. As P^1 strictly decreases, so will $u^c(\rho_1)$'s wage.

It follows immediately from Equation (35) and Proposition 6 that sector two wages raise strictly more for more talented agents. Additionally, the wage of the previously worst agent will increase by more than the lowest wage (as $\Delta_\rho v^c(\rho) \leq 0$): these two facts imply that top sector two wages will increase more than the lowest wage – and the increase in top sector one wages is even greater (by Equation (36)), which completes the proof of (iii). All results that refer to a strictly positive mass of agents follow from the continuity of wage functions. □

Comparing Proposition OA1 with Proposition 4, it becomes clear that as we isolate the effects of the spread from the effects of the improvement in levels, the results become very similar to the scarce jobs case.

Isolated Improvement in Levels

Definition OA2. A rise in sector one surplus is *strict and isolated* if the old and new surpluses are spreads of each other and:

$$\pi^1(u^c(\rho_1), P^1(\rho_1), \rho_2) > \pi^1(u^c(\rho_1), P^1(\rho_1), \rho_1).$$

A strict and isolated rise in surplus levels holds spread constant whilst increasing the surplus for all agents: and thus also for agents with talent $u^c(\rho_1)$. Therefore, it is a special case of a simultaneous increase in spread levels and surplus. This implies that a strict and isolated rise in surplus levels increases the supply of talent in sector one and decreases it in sector two. However, neither of these changes is necessarily strict. An isolated improvement does not change how differentiated talents are, and hence it results in agents' relocation only if the demand for sector one talent shifts up at the extensive margin. This happens only if some additional sector one firms become competitive – so if some of them were not matched before ($M^1(\rho_1) < R^1$) and the increase in skill is high enough to make them more productive than the previously least productive sector two firms ($\pi^1(u^c(\rho_1), P^1(\rho_1), \rho_2) > \pi^2(v^c(\rho_1), P^2(\rho_1))$).

Again, I focus on cases where $R^1, R^2 \geq 1$, as well as assume that the surplus function in sector two is strictly supermodular. The former implies that strict and isolated increases in surplus levels do cause some additional sector one firms to enter the market.

Proposition OA2. If jobs are abundant in each sector, then a strict and isolated rise in surplus levels results in a strict increase in sector one talent supply and strict fall in sector two talent supply.

Proof. It suffices to show that the strong impossibility property (see Definition 15) holds, as real spread implies an increase in vertical differentiation and then the result follows from Theorem 2. Suppose not, then $\Delta_\rho M^2(\rho) \geq 0$ and $\Delta_\rho v^c(\rho) \leq 0$, which implies that $\Delta_\rho M^1(\rho) \leq 0$, $\Delta_\rho u^c(\rho) \geq 0$ and trivially $\Delta_\rho P^2(\rho) \leq 0$ and $\Delta_\rho P^1(\rho) \geq 0$. This and Equations (13)-(14) give us:

$$\pi^1(u^c(\rho_1), P^1(\rho_1), \rho_1) \geq \pi^1(u^c(\rho_2), P^1(\rho_2), \rho_2),$$

which together with the isolated, strict improvement implies:

$$\pi^1(u^c(\rho_1), P^1(\rho_1), \rho_2) > \pi^1(u^c(\rho_2), P^1(\rho_2), \rho_2).$$

This is impossible as $\pi^1_u > 0$ and $\pi^1_h \geq 0$. □

The value added of this result is that with abundant jobs, an improvement in surplus levels may cause some agents to relocate even if marginal surplus remains unchanged.

Proposition OA3. If jobs are abundant in each sector, then a strict and isolated rise in surplus levels (i) strictly increases wages in both sectors, for all talent levels (ii) decreases the critical ability level in sector one and increases in sector two, with at least one of these changes being strict.

Proof. (ii) As marginal surplus is unchanged, any potential differences in wage inequality are driven entirely by reallocation. Note that Equation 7 implies that the critical abilities need to move in opposite directions. Suppose that $u^c(\rho_2) \geq u^c(\rho_1)$ and $v^c(\rho_1) \geq v^c(\rho_2)$. Then, by the same reasoning as in the proof of Proposition OA1, the difference between $w^2(v^*)$ and C^2 would increase; and the inverse of this argument implies that the difference between $w^1(u^*)$ and C^1 would decrease. As, $C^1 = C^2$, this implies that $w^1(u^*)$ increases by less than $w^2(v^*)$. This, however, contradicts the fact that more top workers join sector one (specifically Equation (36)). Therefore, it has to be the case that $\Delta_\rho u^c(\rho) \leq 0$ and $\Delta_\rho v^c(\rho) \geq 0$, with at least one inequality holding strictly.

(i) The increases in sector two and top sector one wages will be strict, as the increase in v^c implies that $v^c(\rho_2)$ will either produce strictly higher surplus (if $v^c(\rho_2) = v^c(\rho_1)$, by Proposition OA2) or receive strictly more of (at least) the same amount of produced surplus (if $v^c(\rho_2) > v^c(\rho_1)$). Moreover, for isolated, strict increases in surplus level, it has to be the case that all sector one wages increase and thus all agents are better off. To see this, note that the difference between wages of any two agents decreases, as the spread of surplus is constant and all agents are matched with less productive firms (Equation 35)). Thus, we have that $\Delta_\rho w^1(u,\rho) > \Delta_\rho w^1(u^*(\rho_1),\rho) > 0$ for all $u \geq u^c(\rho_1)$. □

In sector one the direct upward shift in demand for talent (at the extensive margin) dominates the general equilibrium effect of increased talent supply. In sector two, there is only the general equilibrium effect of decreased supply present, which results in higher wages. Thus, all agents benefit from an increase in skill levels that does not affect spread.

At first glance, a strict and isolated rise in surplus levels seems to create a contradiction as far as wage inequality is concerned. On one hand, the increase (fall) in talent supply in sector one (two) implies that wages will raise less (more) for top than bottom agents; on the other hand, the difference between top and lowest wage cannot fall in sector one if it increases in sector two. This not-quite-contradiction is caused by the fact that although all agents are better off in absolute terms, some occupy a different relative position than previously. In particular, the agents who used to earn lowest wages in sector one will not do so any more: and the fact that they will gain more than the top agents, does not mean that the lowest wage will increase more than the top ones.

Population-wise, the direction of change in wage range is ambiguous. To simplify exposition, I show why is this the case for multiplicative sector one surplus. Note that for this surplus function any strict and isolated improvement in levels is equivalent to a shift of X_1 by some positive constant.

Proposition OA4. Consider some random variable Q with upper lower bound equal to zero. Suppose that in sector one: (i) firms' productivity is strictly positive, (ii) surplus is multiplicative ($\Pi^1(x,z) = xz$) and (iii) the skill distribution is a righ-shift of Q ($X_1(\rho) = Q + \rho$, where $\rho \geq 0$). Then there exists a unique range-maximising right-shift of Q, denoted as $X_1(\rho_c)$. Moreover, the gap between the globally highest and lowest wage increases strictly for $\rho \in [0, \rho^c]$ and decreases strictly for $\rho \in [\rho^c, \frac{\Pi^2(y_h, z_h^2)}{z_l^1}]$.

Proof. First, note that for any $\rho \in [0, r_h]$, where $r_h = \frac{\Pi^2(y_h, z_h^2)}{z_l^1}$, the random variable $X(\rho) = Q + \rho$ results in a matching problem that meets conditions (a)-(d) from Section 2. Moreover, for $\rho = 0$ we have that $\Pi^1(x_l, z_1) \leq \Pi^2(y_l, z_2)$ for any z_1, z_2 and thus $u^c(0) \geq 0$. It follows trivially from Proposition OA2 that there has to exist a unique $\rho_c \geq 0$ such that $u^c = v^c = 0$ and that for any $\rho \leq \rho_c$, $v^c = 0$ and $u^c \geq 0$, whereas for any $\rho \geq \rho_c$, $v^c > 0$ and $u^c = 0$.

Take any $0 \leq \rho_1 < \rho_2 \leq \rho^c$: then as the critical ability in sector two remains constant, wage range in that sector has to be greater for ρ^c than ρ_2, which implies that the same is true for sector one. And similarly, for any $\rho_h \geq \rho_2 > \rho_1 \geq \rho^c$ we have that u^c remains constant and thus wage range in sector one is lower for ρ_2 than ρ_1, which implies that the same is true for sector two. This proves the last statement. The first statement follows from that and the fact that for $\rho > \rho_h$ all agents work in sector one and thus wage inequality is constant. $\square$

To understand this result, note that as the vertical structure of the market is constant – as spread is unchanged – the only way top firms can face more competition for talent is from firms in another industry. As sector one skills improve from very low levels, more sector one firms start competing for high talent. However, as sector one technology keeps improving, eventually it overtakes sector two and some sector two firms cease to compete for top talent, which again increases competition among workers and decreases the top workers' premium.

Market Power and Vertical Differentiation

Firms have no market power, if every firm can be replaced by a more productive company that currently makes zero profits.

Definition OA5. Firms *have no market power* if, in any stable matching and for any matched pair (u, v) (h', i), there exists some $h \neq h'$ such that $\pi^i(u, v, h') \leq \pi^i(u, v, h)$ and $r^i(h) = 0$.

The lack of market power on firms side is equivalent to workers earning the entire surplus – or, alternatively, to the fact that all firms make zero profits.

Lemma OA6. Firms have no market power if and only if, in all stable matchings, all workers earn their entire surplus.

Proof. It follows trivially from Definitions OA5 and 4 that if firms have no market power the all workers earn the entire surplus. I will prove the other part by contradiction. Firstly, note that if all workers earn their entire surpluses, then $r^i(h) = 0$ for all (h, i). Take any $(h', k) \in [0, 1] \times \{1, 2\}$ and any stable matching $\zeta(\bullet)$. Define the set $J = \{(h, i) : \pi^k(\zeta^{-1}(h'), h') \leq \pi^i(\zeta^{-1}(h'), h)$ and $(h, i) \neq (h'k)\}$. If firms have market power, then it has to be the case that for any $(h, i) \in J$, $r^i(h) > 0$. However, this can be only true if J is empty. But then, by continuity, there has to exist some small ϵ such that $\pi^1(\zeta^{-1}(h') + \epsilon, h') > \pi^1(\zeta^{-1}(h') + \epsilon, \zeta(\zeta^{-1}(h') + \epsilon)) = w(\zeta^{-1}(h') + \epsilon) + r(h')$, which contradicts the stability of matching ζ.

□

Trivially, firms have no market power in Roy-like models (see Section 2.3.4). This follows immediately from the fact that in Roy-like models wages are equal to surplus, but can be also shown directly from Definition OA5.

Proposition OA7. If firms have no market power in both the old and new matching problem, then any increase in the levels of surplus in sector i increases the supply of talent in that industry and decreases it in the other sector, regardless of the associated change in the spread of surplus.

Proof. By Lemma OA6 if firms have no market power, then wages are equal to the surplus produced. Therefore, from Equation (5) follows that $\psi(\rho_2) \leq \psi(\rho_1)$ and from Lemma 2 and Equation 7 follows that $u^c(\rho_2) \leq u^c(\rho_1)$ and $v^c(\rho_2) \geq v^c(\rho_1)$. Hence, as $C_{uv} > 0$, the statements of the Proposition follow from Lemma 3 and Definition 9. □

If firms have no market power then skill levels fully determine equilibrium talent supply, regardless of how strong is the vertical differentiation of workers. The reason, as discussed in the main text, is that vertical differentiation of workers determines how much of surplus is received by agents – however, if firms have no market power, then workers earn the entire surplus anyway and their vertical differentiation does not matter.

Omitted Figures

All figures in this section are plotted as if the surplus function provided in the introduction measured surplus in thousands of, say, dollars. Or, equivalently, they

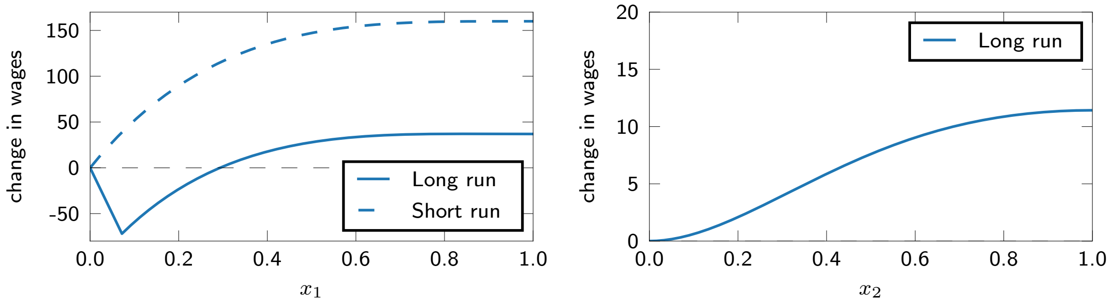

Figure 1: The effect of the surplus shock in finance on wages in financial (left) and production (right) sectors.

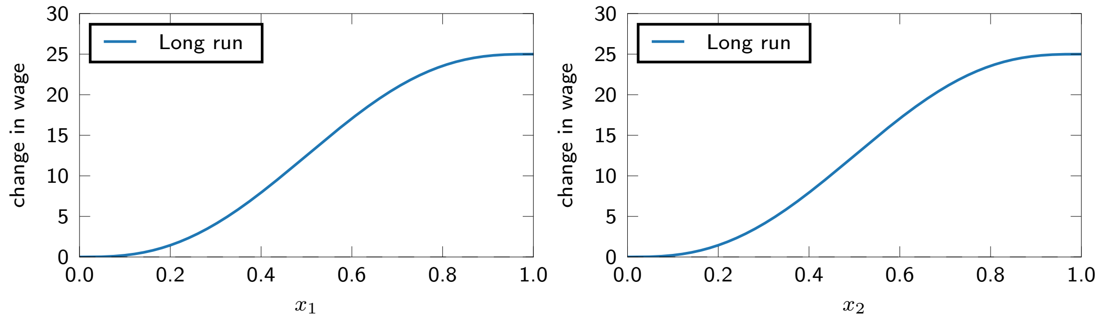

Figure 2: The effect of the overall skill supply shock on wages in financial (left) and production (right) sectors.

are plotted for the following surplus function: $1000(x_i + 1)(z_i + 1)$. Figure 1 depicts the change in wages in response to the first (surplus) shock considered in the introduction. As shown in Proposition 4 wages in the production sector increase for all talent levels – and the change increases in talent. In the financial sector, wages increase for agents with talent greater than 0.293, but fall for agents which are less talented than that. It is worth noting that most of the fall in banking wages that results from re-sorting is due to the change in u^c – as evidenced by the fact that for high enough talent levels the long run change in wages is of the same shape as the short run change.

Figure 2 depicts the change in wages in response to the second (skill supply) shock considered in the introduction. As mentioned there, because of symmetry the effects are exactly the same for production and financial sectors. Wages increase for all skill levels and the change increases in talent. This does not necessarily mean all workers will be better off – as the copula has changed, at least some of them will have a different talent vector than previously.